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Quark mass dependence of hadron spectrum in holographic QCD

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ABSTRACT: We compute a shift of baryon mass spectra due to quark masses in perturbation, in Sakai-Sugimoto model of holographic QCD. We find the shift for the ground state nucleons to be $\delta M = 4.1 \; m_\pi^2 \, \text{GeV}^{-1}$, which is consistent with the current lattice QCD result. We predict the same value of the shift for N(1535) and Δ, while a larger value 7.7 $m_\pi^2 \, \text{GeV}^{-1}$ for Roper N(1440). We also present some evidences that the shifts of the vector meson masses are suppressed in the large 't Hooft coupling limit.

Contents

1. Introduction

Recent progress in holographic QCD of string theory enables us to compute the important physical observables of QCD: the hadron masses. QCD has only two kinds of dimension-ful parameters: the prime one is the QCD scale which basically generates all the hadron masses, and the quark masses. For the light quarks, the quark masses can be treated as a perturbation (while heavy quarks have their particular importance in various phases in QCD). In this short paper, we present a computation of the hadron mass shift due to the quark masses, in the Sakai-Sugimoto model [1, 2] which is the most successful model of holographic QCD so far.

Sakai-Sugimoto model is a holographic dual of *massless* QCD. There proposed two ways to introduce the quark mass to the model: (i) worldsheet instantons [3, 4] (see also [5, 6]), and (ii) tachyon condensation of the D8/$\overline{\text{D8}}$-branes [7]. We are going to use the worldsheet instanton method (i), since the other method (ii) assumes a tachyon "effective" action which is difficult to be validated in any manner in string theory except for using string field theories.*

The worldsheet instanton produces the quark mass term in QCD, and the gravity dual of the worldsheet instanton generates a term of the form

$$\delta L = c \operatorname{tr} [MU] \tag{1.1}$$

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*The tachyon approach has more direct analogue with a famous bottom-up model of holographic QCD [9], and the mechanism is similar to the Higgs mechanism, while the worldsheet instanton approach mimics extended technicolor, see [4, 8].

where M is the quark mass matrix of the form $M =\mathrm{diag}(m_\mathrm{u}, m_\mathrm{d})$, and U is the pion field in the standard notation in the chiral perturbation theory: $U \equiv \exp[2i\pi(x)/f_\pi]$. The value c is computed from the worldsheet instanton amplitude, but it turned out to be difficult to evaluate (for some computations, see [4, 3, 5]). Except for the pion mass through (1.1), the contribution of quark masses to other meson masses has not been computed yet.

In this paper, we present two computations. First, in Sec. 2, we compute shift of the baryon mass spectra due to the quark mass, by assuming that the quark mass dependence in the meson effective lagrangian of the Sakai-Sugimoto model appears only in the new term (1.1), at the leading order in expansion in $1/\lambda$. This assumption will be discussed in Sec. 3. There we will provide some computations and show that shift of the vector/axial vector meson masses is small and at a higher order in $1/\lambda$ expansion.

Under the assumption mentioned above, it is quite easy to compute the baryon mass shift. As noted in [1] and computed in detail in [10],[†] the baryon in the Sakai-Sugimoto model is nothing but an instanton-like soliton in the 5-dimensional Yang-Mills-Chern-Simons (YMCS) action describing all the mesons effectively in a unified way. The term (1.1) can be thought of as a perturbation to the YMCS action, so the mass shift can be computed by just inserting the soliton configuration obtained in [10] to (1.1). The resultant correction is evaluated with baryon wave functions of [10]. Since the parameter c is undetermined, we obtain the baryon mass as a function of the pion mass. Our result for nucleons is in rough agreement with results of lattice QCD. The mass shifts for N(1535) and Δ turn out to be the same as that of the nucleons, while the shift for Roper N(1440) is larger.

2. Baryon Mass Shift

2.1 Set-up in Sakai-Sugimoto model

2.1.1 A review of baryons

The Sakai-Sugimoto model [1, 2] is described by the following five-dimensional $U(N_f)$ YMCS action in a curved background,

$$S = S_\mathrm{YM} + S_\mathrm{CS} , \quad S_\mathrm{YM} = -\kappa \int d^4xdz \; \mathrm{tr}\left[\frac{1}{2}\, h(z)\mathcal{F}_{\mu\nu}^2 + k(z)\mathcal{F}_{\mu z}^2 \right] . \quad (2.1)$$

S_CS is the 5-form Chern-Simons term S_CS. The $U(N_f)$ gauge fields $\mathcal{A}_\mu(\mu = 0, 1, 2, 3)$ and $\mathcal{A}_z$ have their field strength defined in the standard manner, $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu - i[\mathcal{A}_\mu, \mathcal{A}_\nu]$. The extra dimension z unifies all the vector/axial vector mesons via Kaluza-Klein like decomposition of the gauge fields, through the nontrivial functions

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[†]See also [11, 12] for an alternative description of baryons by introducing 5-dimensional fermion fields. Other work for introducing baryons in holographic QCD includes [13].

$h(z) = (1 + z^2)^{-1/3}$ and $k(z) = 1 + z^2$. The constant κ is defined as $\kappa = a\lambda N_c$ with $a = 1/(216\pi^3)$, where λ is the 't Hooft coupling constant. The dimensionful parameter M_{KK} in the action (2.1) is already put to be the unity, and it was argued in [1, 2] that the values $M_{\mathrm{KK}} = 949$ MeV and $\kappa = 0.00745$ can fit the observed values of the ρ meson mass $m_\rho \simeq 776$MeV and the pion decay constant $f_\pi \simeq 92.6$ MeV. In this paper we consider $N_f = 2$, the two-flavor case, and then the $U(2)$ hermitian matrix $\mathcal{A}$ is decomposed by Pauli matrices τ^a ($a = 1, 2, 3$) and a unit matrix $\mathbf{1}_2 \equiv \tau^0$,

$$\mathcal{A} = A + \widehat{A}\frac{\mathbf{1}_2}{2} = A^a\frac{\tau^a}{2} + \widehat{A}\frac{\mathbf{1}_2}{2} . \qquad (2.2)$$

The baryon is identified as a soliton solution localized in the 4-dimensional x^M space ($M = 1, 2, 3, z$) [1]. The instanton number of the Yang-Mills theory (2.1) is identified with the baryon number.

The authors of [10] found an explicit solution of the equations of motion of (2.1). In view of the fact that the size of the baryon scales as $\lambda^{-1/2}$ as found in [11, 10], an appropriate rescaling of the gauge fields and the coordinate by a factor of λ was identified in [10]. But here, for our later purpose, we write the un-rescaled solution. The solution is almost identical to a BPST instanton except for a $1/\lambda$ "correction" appearing in the $U(1)$ part of the zeroth component of the gauge field. This sub-leading contribution can be understood from the fact that the solution represents a baryon whose charge should be identified with the electric charge of the trace of the vector part of the chiral symmetry. The explicit solution is

$$A_M^{\mathrm{cl}} = -if(\xi)g\partial_M g^{-1}, \quad \widehat{A}_0^{\mathrm{cl}} = \frac{1}{\lambda}\frac{1}{8\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2}\right], \quad A_0 = \widehat{A}_M = 0, \quad (2.3)$$

and the non-Abelian part is given by the BPST instanton,

$$f(\xi) = \frac{\xi^2}{\xi^2 + \rho^2} , \quad g(x) = \frac{(z - Z)\mathbf{1}_2 + i(\vec{x} - \vec{X})\cdot\vec{\tau}}{\xi} . \qquad (2.4)$$

Here the moduli parameters of the instanton come in explicitly: the instanton size ρ and the instanton location $X^M = (X^1, X^2, X^3, Z) = (\vec{X}, Z)$. ξ is the distance in this 4-dimensional space, $\xi = \sqrt{(z - Z)^2 + |\vec{x} - \vec{X}|^2}$. In addition to these 5 moduli parameters, the rotation in the gauge $SU(2)$ space are moduli, which are incorporated as $V \in SU(2)$,

$$A_M = VA_M^{\mathrm{cl}}V^{-1} - iV\partial_M V^{-1} . \qquad (2.5)$$

Using the standard technique for quantizing solitons [14, 15], quantized baryon states appear in the degrees of freedom of (Z, ρ, V), a part of which acquires definite potentials in their quantum mechanics via the effect of the curved spacetime and S_{CS}

[10]. For example, the classical size and the location Z of the instanton can be fixed by minimizing the potential as

$$\rho_{\rm cl}^2 = \frac{1}{\lambda}\frac{1}{8\pi^2 a}\sqrt{\frac{6}{5}}\ , \quad Z_{\rm cl} = 0\ , \tag{2.6}$$

so the size of the baryon is $\mathcal{O}(\lambda^{-1/2})$. The spectrum of the baryon can be computed by evaluating the hamiltonian of (Z, ρ, V) with their quantum wave functions.

2.1.2 A review of quark mass

We introduce the quark mass to the Sakai-Sugimoto model by the worldsheet instantons. The method was first introduced in [3] and [4]. In these papers, there are some differences in regularization of the worldsheet instantons, and in this paper we follow [4].

The quark mass can be introduced by joining the D8-brane and the $\overline{\rm D8}$-brane by D6-branes [4]. Once the D8 and the $\overline{\rm D8}$ are joined in this way, it is possible to put a Euclidean string worldsheet whose boundary is defined by the color D4-branes, the D8-brane, the D6-brane and the $\overline{\rm D8}$-brane: these four sets of D-branes form a square, in the middle of which one can put the worldsheet instanton. Since the left-handed and right-handed quarks live at the D8-D4 and $\overline{\rm D8}$-D4 intersections respectively, the worldsheet instanton involves with $\bar{q}_L q_R$ vertex which is nothing but the quark mass operator. The instanton amplitude, which is proportional to $\exp[-A/2\pi\alpha']$ where A is the area of the square, gives the quark mass. Therefore the quark mass can be chosen by tuning the location of the D6-branes and their number.

In the gravity dual, the introduced D6-brane is still a probe D-brane, so one can put the worldsheet instanton in the same manner, but now two of the corners are smeared out by the background curved geometry. The meson excitation described by the gauge fields on the D8-branes induces a worldsheet boundary coupling in the worldsheet instanton amplitude,

$$\delta S = c\int d^4x\,{\rm Ptr}\left[M\left(\exp\left[-i\int_{-z_m}^{z_m}\mathcal{A}_z dz\right] - \mathbf{1}_2\right)\right] + {\rm c.c.,}\quad M \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \tag{2.7}$$

where m_u and m_d are quark masses, and $z = \pm z_m$ is the location of the D6-brane ($z = z_m$ and $z = -z_m$ specify the same radial position, on the D8 and the $\overline{\rm D8}$). This exponential form is a familiar boundary coupling of string worldsheet, and the integral is over the period where the string worldsheet ends on the D8-brane. For nonzero values of the quark mass, the chiral symmetry is explicitly broken, which is consistent with the standard expectation for the quark mass term in QCD. In [4] a rough evaluation of the constant c was given. The constant subtraction $-\mathbf{1}_2$ is for our convenience, to make sure that when $\mathcal{A}_z = 0$ the instanton amplitude vanishes.

The $\mathcal{A}_z$ field in nothing but the pion field [1, 2], and the relation is

$$\mathrm{P}\exp\left[-i\int_{-\infty}^{\infty}dz\mathcal{A}_z\right] = \exp\left[2i\pi(x)/f_\pi\right] \equiv U\ , \tag{2.8}$$

where U is the standard notation for the pion field in the chiral perturbation theory. Using this, the additional action to the Sakai-Sugimoto model (2.1) is

$$\delta S = \int d^4x \, \delta L \, , \quad \delta L \equiv c \, \text{tr} \left[M(U + U^\dagger - 2\mathbf{1}_2) \right] \, , \tag{2.9}$$

which is the well-known term in the chiral perturbation theory. Here although z_m is a finite value, we approximate the exponential part in (2.7) by (2.8). For the purpose of a rough estimate, this simplification can be justified since the pion wavefunction is localized at $z = 0$.

Using the pion mass m_π, we obtain a relation

$$c(m_u + m_d) = \frac{1}{2} f_\pi^2 m_\pi^2. \tag{2.10}$$

We are working in the normalization $\pi(x) = \pi^a(x)T_a$, $\text{tr}[T_a T_b] = \frac{1}{2}\delta_{ab}$.

We have seen that the introduction of the D6-brane ending on the D8 and the $\overline{\text{D8}}$-branes can induce a quark mass term. However, the additional action (2.7) should not be the only correction to the Sakai-Sugimoto action. In fact there are possibly two other sources for new terms coming from the presence of the D6-brane:

- Vector meson / axial vector meson mass terms.

- Shift of D8-brane configuration pulled by the worldsheet instanton.

The first one is very interesting, as it may give an interesting result on shift of the meson mass spectra due to the quark mass. However, in the next section, we are going to discuss that this shift is suppressed by $1/\lambda$ in the holographic QCD. The second one, the shift of the location (shape) of the D8-branes, is intriguing of its own and was studied in [3]. Since this is obviously a secondary effect in the sense that the string worldsheet instanton pulls the heavy D8-branes, we will not consider it in this paper.[‡]

In the following in this section, we will ignore these two additional possibilities and consider only (2.7) for computing the baryon mass shift.

2.2 Baryon mass shift : classical evaluation

When the quark mass is small, at the leading order in the quark mass, the shift of the baryon mass is simply given by

$$\delta M = - \int d^3x \, \delta L[A^{\text{cl}}] \tag{2.11}$$

[‡]This tiny modification of the shape of the D8-brane can be numerically computed, and in fact it increases the mass of baryon. See [16] for a relevant discussion. This will also change the meson masses.

where $A^{\rm cl}$ is the classical solution (2.3) representing the baryon, and δL is given by (2.7). By the additional term (2.7), the baryon configuration itself would be modified and it would shift the baryon mass at the higher order in the quark mass, but we neglect the higher order terms in this paper.

If we substitute the classical solution to (2.7), the resultant expression is a function of the instanton moduli. At the classical level (which is the large N_c limit), we can substitute the classically fixed moduli (2.6). To distinguish different baryon states, we use baryon wave functions of the moduli given in [10] and take the expectation value of (2.7), which gives the baryon mass shift for a given baryon state. Here we shall compute the classical value first, and then in the next subsection we will compute the quantized value for each baryon state.

Let us evaluate this (2.11) explicitly. First, note that in the large λ expansion, the leading order term of the solution is just the same as the BPST instanton, while the term of the next-to-leading order is nonzero only for the zero-th component $\widehat{A}_0(x, z)$ of the overall $U(1)$ part of the gauge field. On the other hand, the integral in defining U in (2.8) which appear in (2.11) is only for the z component. Thus, the sub-leading term does not contribute to our calculation, so what we need is just the BPST instanton part.

Eventually, the integral (2.8) was already evaluated in [17], so let us use it. For simplicity, we put all the moduli to be the classical values, (2.6) and $V = \mathbf{1}_2$, to obtain a classical shift of the baryon mass (we can put $X^a = 0$ without losing generality). We work in a singular gauge of the BPST instanton,

$$A_z = \left(\frac{1}{\xi^2} - \frac{1}{\xi^2 + \rho^2} \right) (x^a - X^a)\tau^a . \tag{2.12}$$

The A_z configuration (2.12) is proportional to a matrix $(x^a)\tau^a$ for any value of z, so the evaluation of the path ordering in (2.11) reduces to an Abelian problem [17]. We obtain

$$U = \exp\left[if(r)\hat{x}^a\tau^a\right] \tag{2.13}$$

with

$$f(r) = \pi \left[1 - \frac{1}{\sqrt{1 + \rho^2/r^2}} \right], \quad r \equiv \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}. \tag{2.14}$$

With this expression, we obtain

$$U + U^\dagger = 2\cos(f)\mathbf{1}_2. \tag{2.15}$$

Therefore, using the relation (2.10), we have

$$\delta M = \int d^3x \; f_\pi^2 m_\pi^2 (1 - \cos(f))$$

$$= 4\pi f_\pi^2 m_\pi^2 \rho^3 \int d\tilde{r} \; \tilde{r}^2 \left(1 - \cos\left[\pi \left(1 - \frac{1}{\sqrt{1 + \tilde{r}^{-2}}} \right) \right] \right) \tag{2.16}$$

where we made a change of a variable as $r = \rho\tilde{r}$. The last integral is numerically evaluated as

$$\int d\tilde{r}\, \tilde{r}^2 \left(1 - \cos\left[\pi\left(1 - \frac{1}{\sqrt{1 + \tilde{r}^{-2}}} \right) \right] \right) = 1.104. \tag{2.17}$$

Using this, we obtain a formula for the shift of the baryon mass spectrum due to the quark mass,

$$\frac{\delta M}{m_\pi^2} = 4\pi f_\pi^2 \rho^3 \times 1.104\ . \tag{2.18}$$

In the classical limit $N_c \to \infty$, ρ is equal to $\rho_{\rm cl}$ which is given in (2.6), so

$$\delta M = 1.104 \times 4\pi f_\pi^2 m_\pi^2 \rho_{\rm cl}^3 = 1.104 \times 4\pi f_\pi^2 m_\pi^2 \left(\frac{27\pi\sqrt{6}}{\sqrt{5}\lambda} \right)^{3/2}. \tag{2.19}$$

In this limit, the mass shift (2.18) is independent of the baryon state, since the dependence on the baryon state appears at the quantum level which is sub-leading in the $1/N_c$ expansion. In the next subsection we compute the $1/N_c$ corrections and the baryon state dependence. It was found that $1/N_c$ corrections are rather large for various static properties of baryons (such as charge radii) in [18] .

Note that our baryon mass shift δM is linear in m_π^2 as seen in (2.18), to the lowest order in m_π. This is in agreement with the lattice QCD results and also with chiral perturbation theories.

2.3 Baryon mass shift : baryon state dependence

The classical evaluation of the baryon mass shift, (2.18), will be corrected once we take into account the quantum states of the baryon. Each baryon state is specified by the quantum numbers $\{I(= J), I_3, n_\rho, n_Z\}$ where I is the isospin, J is the spin, n_ρ and n_Z are quantum numbers associated with the moduli parameters ρ and Z [10]. In general the mass shift depends on these quantum numbers.

The dependence can be evaluated by the standard perturbation in quantum mechanics. The additional hamiltonian is (2.11) evaluated with all the moduli dependence in $A_{\rm cl}$.

First, let us consider Z-dependence. Since the Z-dependence comes in as $z - Z$ in the solution (2.3) with (2.4), once we perform the integration (2.8), Z disappears. So there is no additional hamiltonian for Z.

Next, let us consider V. This $SU(2)$ rotation V appears in the solution as a gauge transformation of the solution, (2.5), the transformation of U is given as VUV^{-1}, so $U + U^\dagger \to V(U + U^\dagger)V^{-1}$. Now, looking at (2.15), we notice that in fact this V dependence disappear, because $U + U^\dagger$ is proportional to the unit matrix. So, there is no additional hamiltonian for V.

Finally, let us consider the moduli ρ. The expression (2.18) has a factor ρ^3, so it is the additional hamiltonian for the quantum mechanics for ρ. In sum, the shift of the baryon mass is dependent only on the quantum number n_ρ. It is not sensitive to spin/isospin and also the quantum number n_Z. This means that, in particular, proton, neutron, $N(1535)$ and delta excitation have the same mass shift since they share the same value $n_\rho = 0$, while that of the Roper excitation $N(1440)$ ($n_\rho = 1$) is different. In particular, it is interesting that, although generic values of (m_u, m_d) seem to break the isospin invariance, our result shows that the leading order mass shift of the baryons is insensitive to this breaking.

We need to evaluate $\langle \rho^3 \rangle$ with the baryon wave function $R_{n_\rho}(\rho)$. The explicit expression for the wave function is given in [10]. For the lowest $n_\rho = 0$, it is

$$R_0(\rho) = \mathcal{N}_0 \, \rho^{-1+2\sqrt{1+N_c^2/5}} \exp\left[-\frac{M_0}{\sqrt{6}} \rho^2 \right] \tag{2.20}$$

where $M_0 \equiv 8\pi^2 \kappa$, and $\mathcal{N}_{n_\rho}$ is the normalization factor. The wave function is normalized as

$$\int d\rho \, \rho^3 R_{n_\rho}(\rho)^2 = 1 , \tag{2.21}$$

with the factor ρ^3 which is the Jacobian for the 4-dimensional spherical coordinate system in the one-instanton moduli space. The expectation value is evaluated as

$$\langle \rho^3 \rangle_{n_\rho=0} = \frac{\displaystyle\int_0^\infty d\rho \, \rho^6 R_0(\rho)^2}{\displaystyle\int_0^\infty d\rho \, \rho^3 R_0(\rho)^2} = \left(\frac{\sqrt{6}}{2M_0} \right)^{3/2} \frac{\displaystyle\int_0^\infty dt \, t^{3/2+2\sqrt{1+N_c^2/5}} e^{-t}}{\displaystyle\int_0^\infty dt \, t^{2\sqrt{1+N_c^2/5}} e^{-t}}$$

$$= \rho_{\rm cl}^3 \left(\frac{\sqrt{5}}{2N_c} \right)^{3/2} \frac{\Gamma(b+3/2)}{\Gamma(b)} , \tag{2.22}$$

where in the second equality we rescaled the variable as $\rho = (\sqrt{6}/(2M_0))^{1/2} t^{1/2}$, and we have defined $b \equiv 1 + 2\sqrt{1 + N_c^2/5}$. The numerical value of (2.22) for $N_c = 3$ is found as

$$\langle \rho^3 \rangle_{n_\rho=0} / \rho_{\rm cl}^3 = 2.23 . \tag{2.23}$$

Using (2.23), $M_{\rm KK} = 949$ [MeV] and $\kappa \equiv a\lambda N_c = 0.00745$ which were used in [1, 2][§], we can evaluate the mass shift (2.18) for the baryon states with $n_\rho = 0$. We obtain the value of the baryon mass shift as[¶]

$$\frac{\delta M_{n_\rho=0}}{m_\pi^2} = 4.11 \ [\text{GeV}^{-1}] . \tag{2.24}$$

[§]This set of values was obtained by fitting $f_\pi = 92.6$ [MeV] and the ρ meson mass in [1].

[¶]One could use instead the value $M_{\rm KK} = 500$ [MeV] which was used in [10] for fitting the absolute values of the baryon masses, but then it causes an ambiguity on the choice of observable quantities one makes for getting the value of λ.

In particular, our result is universal for all spin, isospin and n_Z.

We can compare our result (2.24) for nucleons ($n_\rho = 0$) with numerical results obtained in lattice QCD simulations. For example, a lattice result for nucleons in [19] shows $\delta M_{\text{nucleon}}/m_\pi^2 = 4 \times 1.02(7)$ [GeV^{-1}], and a summary of lattice results for nucleons in [20] is $\delta M_{\text{nucleon}}/m_\pi^2 = 4 \times 0.9$ [GeV^{-1}]. Ref. [21] provides a summary of lattice QCD results in the year of 2003, and shows values of the coefficient as $4 \times (0.93(5), 1.25(5), 0.93(4), 1.11(4))$ [GeV^{-1}] for four different fitting schemes with chiral perturbation theory. Rather recent results of lattice QCD [22, 23] show similar values. All of these values are consistent with ours (2.24).[||]

Δ excitation share the same quantum number $n_\rho = 0$, thus our value (2.24) also applies to Δ. A lattice result can be found in Ref. [24] as $4 \times 1.20(5)$ for Δ^{++} and $4 \times 1.19(8)$ for Δ^+, which are again consistent with ours (2.24).

For $n_\rho = 1$, the baryon wave function is

$$R_1(\rho) = \mathcal{N}_1 \left(\frac{2M_0}{\sqrt{6}} \rho^2 - 1 - 2\sqrt{1 + N_c^2/5} \right) \rho^{-1+2\sqrt{1+N_c^2/5}} \exp\left[-\frac{M_0}{\sqrt{6}} \rho^2 \right] . \quad (2.25)$$

In the same manner, we obtain

$$
\begin{aligned}
\left\langle \rho^3 \right\rangle_{n_\rho=1} &= \left(\frac{\sqrt{6}}{2M_0} \right)^{3/2} \frac{\int_0^\infty dt \left(t - 1 - 2\sqrt{1+N_c^2/5} \right)^2 t^{3/2+2\sqrt{1+N_c^2/5}} e^{-t}}{\int_0^\infty dt \left(t - 1 - 2\sqrt{1+N_c^2/5} \right)^2 t^{2\sqrt{1+N_c^2/5}} e^{-t}} \\
&= \rho_{\text{cl}}^3 \left(\frac{\sqrt{5}}{2N_c} \right)^{3/2} \frac{\Gamma(b+7/2) - 2b\Gamma(b+5/2) + b^2\Gamma(b+3/2)}{\Gamma(b+2) - 2b\Gamma(b+1) + b^2\Gamma(b)} \\
&= \rho_{\text{cl}}^3 \left(\frac{\sqrt{5}}{2N_c} \right)^{3/2} \frac{\Gamma(b+5/2) + \frac{9}{4}\Gamma(b+3/2)}{\Gamma(b+1)} , \quad (2.26)
\end{aligned}
$$

which results in a relation for $N_c = 3$,

$$\left\langle \rho^3 \right\rangle_{n_\rho=1} / \rho_{\text{cl}}^3 = 4.16 . \quad (2.27)$$

Using this, the mass shift for the baryon states with $n_\rho = 1$ is obtained as

$$\frac{\delta M_{n_\rho=1}}{m_\pi^2} = 7.65 \text{ [GeV}^{-1}] . \quad (2.28)$$

[||]However, note that our result is a large N_c expansion with a sub-leading order term included (for distinguishing baryon states), and also obtained under the assumptions which are listed in Sec.2.1.2. Since the $1/N_c$ corrections are not so small, it is better to think of (2.24) as just an order estimate of the baryon mass shift.

This is our prediction, stating that the baryon mass shift is larger for the $n_\rho = 1$ states such as Roper excitation, while for the states with $n_\rho = 0$ such as N(1535) and Δ the shift is the same as that of nucleons.[**]

3. Discussions on Meson Mass Shift

The shift of the baryon mass spectrum studied in the previous section relies on an assumption that in the large λ expansion the term (2.7) is the leading order term as for the effect of the quark mass term. This term (2.7) generates a pion mass term as seen in Sec. 2.1.2, and the assumption is equivalent to the statement that the shift of the vector meson mass spectrum is at a higher order in the large λ expansion. In this section, we give some arguments for the assumption. The computations given here are not sufficient to prove the assumption, but may provide some physical intuition on how plausible the assumption is.

We give two arguments. The first one is based on the chiral perturbation theory and its correspondence in string theory amplitudes. The second one is a contribution coming from the D6-brane which we introduced to obtain a finite quark mass from the worldsheet instanton (see Sec. 2.1.2).

3.1 Chiral perturbation and its stringy realization

As seen in Sec. 2.1.2, the string worldsheet instanton induces a term (2.9) of the form $c\,\mathrm{tr}[MU]$, which is nothing but the term for the quark mass perturbation appearing in the chiral perturbation theory. It is the leading term in the derivative expansion. In this subsection, we study other possible corrections to the Sakai-Sugimoto effective action (2.1), from the viewpoint of chiral perturbation theory.

Sakai-Sugimoto model includes vector mesons as massive gauge fields for hidden local symmetries, and then the covariant derivative is derived as

$$D_\mu U = \partial_\mu U - iA^L_\mu(x)U + iUA^R_\mu(x) , \qquad (3.1)$$

and the gauge fields $A^L(x) = \mathcal{A}(x, z = -\infty), A^R(x) = \mathcal{A}(x, z = +\infty)$ transform as

$$A^L_\mu(x) \ \rightarrow ig^L(x)\partial_\mu g^L(x)^{-1} + g^L(x)A_\mu g^L(x)^{-1} , \quad g^L(x) = g(x, z = -\infty) \quad (3.2)$$

$$A^R_\mu(x) \rightarrow ig^R(x)\partial_\mu g^R(x)^{-1} + g^R(x)A_\mu g^R(x)^{-1} , \quad g^R(x) = g(x, z = +\infty) , (3.3)$$

[**]It was noted in [10] that, in the baryon spectrum of massless QCD, there is a degeneracy in the mass spectrum for states with common value of the sum $n_\rho + n_Z$. In view of our results, this degeneracy is resolved once quark masses are turned on. Our results suggest that the mass shift of the $(n_\rho, n_Z) = (1, 0)$ state (Roper $N(1440)$) is larger than that of the $(0, 1)$ state $N(1535)$ (which is parity odd). However, in reality the observed mass of the Roper is lighter than that of $N(1535)$. This may indicate that the observed resolution of the degeneracy in the mass spectrum is not by the quark mass but rather would be by some higher order corrections in $1/N_c$ and $1/\lambda$ in massless holographic QCD.

under the gauge transformation $g(x, z) \in SU(N_f)$ on the D8-branes.

Then we can write a higher order term which appears in the sense of chiral perturbation theory as

$$\frac{1}{f_\pi^2} \text{Tr}[MUD_\mu U^\dagger D^\mu U] \ . \tag{3.4}$$

This is, in the same manner as $\text{tr}[MU]$), invariant under the global part of the gauge transformation, if we simultaneously rotate the quark mass matrix M, according to the philosophy of the chiral perturbation. In the chiral perturbation, this is the leading correction to the $\text{tr}[MU]$ term in derivative expansion.[††] This term in fact generates (axial) vector meson mass terms. It is obvious if once we expand the gauge field $\mathcal{A}_\mu$ in terms of the KK-decomposed states which are nothing but the vector meson states.

Now, our concern is the order of the coefficient of this term. In the evaluation of the baryon mass shift in the previous section, we simply neglected this term. As we will see, we can naively argue that this term is at a higher order in the $1/\lambda$ expansion.

A stringy interpretation of this term can be found as follows. If we think of this (3.4) as a product of MU and $D_\mu U^\dagger D^\mu U$, the former comes from the worldsheet instanton amplitude, while the latter is identical to the pion kinetic term which has been obtained in [1, 2] from the 5-dimensional YMCS kinetic term. So, we can conclude that this (3.4) comes from a worldsheet instanton amplitude with fluctuations of the gauge fields on the D8-branes. More precisely, two vertex operators for the Yang-Mills field on the D8-brane are inserted at the boundary of the worldsheet instanton.

To see the order of this worldsheet instanton amplitude, let us briefly see a situation in the flat background. Worldsheet disk amplitudes in flat spacetime are of the form

$$A \sim \frac{1}{g_s} \int DX \exp\left[\frac{1}{2\pi\alpha'} \int d^2\sigma \partial X \bar{\partial} X - i \oint_{\text{boundary}} d\sigma A[X](dX/d\sigma)\right] \ . \tag{3.5}$$

The gauge fields here are Taylor-expanded as [25]

$$A[X](dX/d\sigma) = A_M[x](dX^M/d\sigma) - \frac{1}{2}F_{MN}[x]\widetilde{X}^N(dX^M/d\sigma) + \cdots \tag{3.6}$$

which is a derivative expansion, and we decomposed X as $X = x + \widetilde{X}$ where x is the zero mode of $X[\sigma]$. The path integral can be explicitly done to this order because the action is quadratic, and the result is nothing but the Born-Infeld action,

$$A \sim \frac{1}{g_s}\sqrt{-\det(\eta + 2\pi\alpha' F)} \ . \tag{3.7}$$

[††] The term $\text{Tr}[MD_\mu D^\mu U]$ is at the same order, but it is related to (3.4) by the equation $U\partial_\mu U^\dagger \partial_\mu U \sim \partial_\mu \partial^\mu U$ which can be obtained by a trivial identity $\partial[U^\dagger U] = 0$.

It can be expanded again as

$$A \sim \frac{1}{g_s}\left[1 + (\alpha')^2 F^2 + \cdots\right] . \tag{3.8}$$

When the disk spans a certain region to give the worldsheet instanton, the zero mode x dependence in the semi-classical saddle point in the path-integral generates the worldsheet instanton factor $\exp[-[\text{area}]/(2\pi\alpha')]$ in front of (3.8).

Now let us try to apply this computation to our case. In our case, we find it difficult because the background spacetime is curved, and also because the worldsheet boundary is on the D8-branes which are curved and joined. (See [5] (and also [26]) for detailed calculations in a supersymmetric spacetime background.) However, we can argue the order of magnitude qualitatively as follows. First, notice that the term "1" in (3.8) in fact corresponds to our leading order term (2.7). The reason why we have the path-ordered factor like an Wilson loop in (2.7) is in fact the first term on the right hand side of (3.6). On the other hand, if this (3.8) is not for a worldsheet instanton but a standard disk amplitude (*i.e.* without winding some D-branes), the F^2 term is nothing but $D_\mu U^\dagger D^\mu U$, as has been found in [1]. Combining these two facts, we come to the conclusion that, compared to the term MU, the term $MUD_\mu U^\dagger D^\mu U$ is suppressed by $(\alpha')^2$.

Since we are in the warped spacetime, this α' should be replaced by an "effective α'" which is given by the tension of a string at at the tip ($z = 0$) of the geometry.* This is the QCD string tension,

$$\frac{1}{2\pi\alpha'_{\text{eff}}} = \frac{1}{9\pi}\lambda M_{\text{KK}}^2. \tag{3.9}$$

In the Sakai-Sugimoto model, higher derivative corrections in string theory appears with this effective α' (see for example discussions in [27]). This argument shows that the term of our concern is of the form

$$c\,\text{tr}\left[MU\left(1 + \mathcal{O}\left(\lambda^{-2}\right)D_\mu U^\dagger D^\mu U\right)\right] , \tag{3.10}$$

corresponding to (3.8). In the large λ expansion which the holographic QCD employs, the term $\text{tr}[MUD_\mu U^\dagger D^\mu U]$ is found to be suppressed by λ^{-2}.

This means that (axial) vector meson masses are not so sensitive to the quark masses, compared to the pion mass shift.

3.2 Effects of D6-branes

In [4], to regularize the area of the worldsheet instanton, D6-branes ending on the flavor D8-branes were introduced. This additional ingredient may affect the meson spectra, and in this subsection we discuss the possible effects of the D6-branes.

---

*Although the gauge fields on the D8-branes are not only at the tip, the wave functions of the vector mesons on the D8-branes is localized at the tip. So we just assume that the effective worldsheet propagator in this background geometry may be approximated by the propagator there.

Note that if one uses a procedure of renormalization of the worldsheet instanton proposed in [3], there is no need for the D6-branes, so nothing is worried about concerning the content of this subsection. On the other hand, the D6-branes in [4] can be thought of as a physical cut-off which not only serves as a physical renormalization point but also provides modes on the D6-branes. Therefore, schematically, the effective lagrangian of the meson sector in total should be

$$S_{\text{D8}} + S_{\text{D6}} + S_{\text{inst}}. \tag{3.11}$$

In the previous subsection we considered S_{inst}, and here let us show that the term S_{D6} is sub-leading in $1/\lambda$ expansion. This S_{D6} is nothing but the D6-brane effective action, which is

$$S_{\text{D6}} = -\mathcal{T}_{\text{D6}} \int d^7x \, e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} \,. \tag{3.12}$$

Here the tension of the D6-brane is given by $\mathcal{T}_{\text{D6}} = 1/(2\pi)^6 l_s^7 g_s$, and the dilaton in the background is $e^{-\phi} = (R/U)^{3/4}$, with the integral $d^7x = d^4x d\tau d\Omega_2$. Note that the D6-brane is along the compactified S^1 direction parameterized by τ which has a period $2\pi/M_{\text{KK}}$.

In the following, we assume that the D6-brane connects the D8 and the anti D8-brane straightly without bending. It is expected that the shape of D6 is not straight in τ but bents, as in the case of the curved shape of the D8-brane. However, for simplicity in this paper we assume the straight shape of the D6-brane.

In addition, in the following computation, we assume that the D6-brane wraps the largest S^2 in the S^4. As studied in an appendix of [4], the shape of the D6-brane in the U-S^4 space transverse to the N_c D4-branes is non-trivial. However, since our purpose here is to estimate the order of magnitude of the D6-brane effect, we may take the simple D6-brane shape.

Expanding the D6-brane action (3.12) to quadratic order in the field strength F and substituting the background metric and dilaton, we obtain

$$S_{\text{D6}} = \frac{-1}{(2\pi)^6 l_s^7 g_s} \int d^4x d\tau d\Omega_2 \, R^{3/2} (U_{\text{D6}})^{1/2} (2\pi\alpha')^2 \left(\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}f(U_{\text{D6}})F_{\tau\mu}^2 \right) \tag{3.13}$$

where $f(U) = 1 - U_{\text{KK}}^3/U^3$, and U_{D6} is the location of the D6-brane in the radial direction of the background. Note that the D6-brane is assumed to be straight and wrap the S^2, so it is specified by just the coordinate $U = U_{\text{D6}}$. The action depends only on the dimensionless ratio $U_{\text{D6}}/U_{\text{KK}}$. Introducing a new variable $u \equiv U/U_{\text{KK}}$, we find that the action of the D6-brane, located at $u_{\text{D6}} = U_{\text{D6}}/U_{\text{KK}}$, is given by

$$S_{\text{D6}} = \frac{-1}{(2\pi)^6 l_s^7 g_s} \int d^4x d\tau d\Omega_2 \, R^{3/2} (U_{\text{KK}})^{1/2} (2\pi\alpha')^2 u_{\text{D6}}^{1/2} \left(\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(1 - u_{\text{D6}}^{-3})F_{\tau\mu}^2 \right) \,.$$

Let us substitute the dictionary [1] for the correspondence between the parameters of the supergravity background and the QCD parameters, $R = (g_{\mathrm{YM}}^2 N_c l_s^2 / 2 M_{\mathrm{KK}})^{1/3}$, $U_{\mathrm{KK}} = (2/9) g_{\mathrm{YM}}^2 N_c M_{\mathrm{KK}} l_s^2$ and $g_s = g_{\mathrm{YM}}^2 / 2\pi M_{\mathrm{KK}} l_s$. Then we get

$$S_{\mathrm{D6}} = -\frac{1}{6\pi^2} N_c M_{\mathrm{KK}} \int d^4 x d\tau \, u_{\mathrm{D6}}^{1/2} \left(\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2}(1 - u_{\mathrm{D6}}^{-3}) F_{\tau\mu}^2 \right) . \qquad (3.14)$$

Note that all the l_s dependence has gone.

The mode expansion of this action for eigen functions of τ gives a spectrum of particle modes living on the D6-brane. This is similar to the analysis of flavor D-brane fluctuations. On the other hand, since the D6-brane ends on the D8-brane, the wave functions of the modes on the D8-branes (which are mesons) can soak into the D6-branes, thus have overlap with the D6-brane eigenmodes. We need to look at this interaction and diagonalize the quadratic action to see the possible effects on the meson mass shift.

First, let us consider the modes living on the D6-brane. Fourier-decomposing the gauge fields along the τ direction gives rise to a mass spectrum with a spacing M_{KK}. The eigen functions are trigonometric functions of $M_{\mathrm{KK}}\tau$, so the integral $\int d\tau$ in (3.14) cancels the factor M_{KK} in front of the action (3.14). This means that properly normalized fluctuations $C_\mu(x)$ with canonical kinetic terms is

$$A_\mu \sim \sum_k N_c^{-1/2} \cos(k M_{\mathrm{KK}}\tau) \, C_\mu^{(k)}(x) \qquad (3.15)$$

up to a numerical coefficient. The mixing can be evaluated later with this proper normalization.

Next, we look at the meson modes (on the D8-branes) which soak into the D6-branes. We have two kinds of mesons: vector mesons whose eigen functions $\psi_{2n}(z)$ are even in z, while axial vector mesons whose eigen functions $\psi_{2n-1}(z)$ are odd in z. Since the D6-branes connect two points on the D8-branes, $z = z_m$ and $z = -z_m$, so depending on the parity of the eigen functions in z, the soaking into the D6-brane would be different. For the vector mesons, the most natural soaking would be just a constant mode on the D6-brane, while for the axial vector mesons, the natural one which costs smallest energy would be $\cos(M_{\mathrm{KK}}\tau)$ on the D6-brane. Once we substitute these soaking functions to (3.14), we obtain

$$S_{\mathrm{D6}} = \frac{-N_c}{6\pi} \int d^4 x \, u_{\mathrm{D6}}^{1/2} \left[\frac{1}{4}(F_{\mu\nu}^{(\mathrm{even})}(z = z_m))^2 \right.$$
$$\left. + \frac{1}{8}(F_{\mu\nu}^{(\mathrm{odd})}(z = z_m))^2 + \frac{1}{2} M_{\mathrm{KK}}^2 (1 - u_{\mathrm{D6}}^{-3})(A_\mu^{(\mathrm{odd})}(z = z_m))^2 \right] \quad (3.16)$$

Here, we divided the wave functions into the odd part and the even part,

$$A_\mu(x, z) = A_\mu^{(\mathrm{odd})} + A_\mu^{(\mathrm{even})}, \qquad (3.17)$$

$$A_\mu^{(\mathrm{odd})} = \sum_{n=1}^\infty B_\mu^{(2n)}(x)\psi_{2n}(z), \quad A_\mu^{(\mathrm{even})} = \sum_{n=1}^\infty B_\mu^{(2n-1)}(x)\psi_{2n-1}(z). \qquad (3.18)$$

Note that to obtain the expression (3.16) we identified the gauge fields on the D6-brane with that of the D8-branes. The reason why there is no difference in the normalizations is as follows. Let us consider a Dp-brane ending on a D$(p+3)$-brane. The Dp-brane can be thought of as a spike solution of a BPS Dirac monopole on the D$(p+3)$-brane [28]. The A_μ ($\mu = 0, \cdots, p$) fluctuations on the D$(p+3)$-brane can soak onto the Dp-brane. The DBI action of the D$(p+3)$-brane is, if we turn off the other fluctuations,

$$
S_{p+3} = \mathcal{T}_{\mathrm{D}(p+3)} \int d^{p+4}x \sqrt{\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} \sqrt{\det(\delta_{ij} + \partial_i X^{(c)} \partial_j X^{(c)} + 2\pi\alpha' F_{ij}^{(c)})}
$$

where $X^{(c)}$ and $A_i^{(c)}$ are classical solution for the spike, and $i, j = p+1, p+2, p+3$. Explicit substitution of this classical solution gives

$$
\mathcal{T}_{\mathrm{D}(p+3)} \int dx^{p+1} dx^{p+2} dx^{p+3} \sqrt{\det(\delta_{ij} + \partial_i X^{(c)} \partial_j X^{(c)} + 2\pi\alpha' F_{ij}^{(c)})} = \mathcal{T}_{\mathrm{D}p} , \quad (3.19)
$$

therefore we obtain

$$
S_{p+3} = \mathcal{T}_{\mathrm{D}p} \int d^{p+1}x \sqrt{\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} \tag{3.20}
$$

which is the standard Dp-brane action. This shows that the gauge field on the D8-brane is in fact a gauge field on the D6-brane, without changing its normalization.

The normalization of the eigen functions $\psi_n(z)$ is defined in [1] as

$$
\frac{g_{\mathrm{YM}}^2 N_c^2}{216\pi^3} \int dz K^{-1/3} \psi_n(z) \psi_m(z) = \delta_{nm}. \tag{3.21}
$$

So, defining the normalized eigen function

$$
\tilde{\psi}_n(z) \equiv \sqrt{\frac{g_{\mathrm{YM}}^2 N_c^2}{216\pi^3}} \psi_n(z), \tag{3.22}
$$

we have the standard normalization $\int dz K^{-1/3} \tilde{\psi}_n(z) \tilde{\psi}_m(z) = \delta_{nm}$. Substituting this normalization to the D6-brane action, we obtain

$$
S_{\mathrm{D6}} = \frac{-9\pi^2 u_{\mathrm{D6}}^{1/2}}{\lambda} \sum_{n,m=1}^{\infty} \tilde{\psi}_{2n-1}(z_m) \tilde{\psi}_{2m-1}(z_m) \int d^4x \left[\frac{1}{4} F_{\mu\nu}^{(2n-1)} F_{\mu\nu}^{(2m-1)} \right]
$$
$$
+ \frac{-9\pi^2 u_{\mathrm{D6}}^{1/2}}{\lambda} \sum_{n,m=1}^{\infty} \tilde{\psi}_{2n}(z_m) \tilde{\psi}_{2m}(z_m) \int d^4x \left[\frac{1}{8} F_{\mu\nu}^{(2n)} F_{\mu\nu}^{(2m)} + \frac{1}{2} M_{\mathrm{KK}}^2 (1 - u_{\mathrm{D6}}^{-3}) B_\mu^{(2n)} B_\mu^{(2m)} \right].
$$
$$
\tag{3.23}
$$

Here F are the field strengths of the (axial) vector mesons $B_\mu(x)$.

The information we need to extract from the last expression (3.23) is its overall factor $1/\lambda$. The action (3.23) is supposed to be added to the D8-brane action (which is the meson action of [1]), so we find a $1/\lambda$ correction due to the D6-brane.

There is an overlap between these soaking modes of the mesons and the modes on the D6-brane (3.15). Since the former has a normalization factor $\lambda^{-1/2}N_c^{-1/2}$ as seen in (3.22) while the latter (3.15) has $N_c^{-1/2}$, substituting them to (3.14), we find the overlap giving a quadratic interaction of order $\lambda^{-1/2}$.

Therefore, in total, we obtain the kinetic terms for the modes on the D8-branes (mesons) and the modes on the D6-brane schematically written as

$$S \sim \int d^4x \, (F^{\mathrm{D8}}, F^{\mathrm{D6}}) \begin{pmatrix} 1 + \mathcal{O}(\lambda^{-1}) & \mathcal{O}(\lambda^{-1/2}) \\ \mathcal{O}(\lambda^{-1/2}) & 1 \end{pmatrix} \begin{pmatrix} F^{\mathrm{D8}} \\ F^{\mathrm{D6}} \end{pmatrix} . \qquad (3.24)$$

Diagonalizing this matrix, we obtain a shift to the vector meson mass which is of order $\mathcal{O}(\lambda^{-1/2})$. As advertised, it is a sub-leading order term, compared to the original kinetic terms in the D8-brane action for the vector mesons. Furthermore, this shift is further suppressed by the wave function factor $\tilde{\psi}(z_m)$.

4. Summary

In this paper, we computed the shift of the baryon mass spectrum by varying the quark mass, from the chiral limit, in the Sakai-Sugimoto model [1, 2] of holographic QCD. To introduce the quark mass to the model, we employed the worldsheet instanton approach [3, 4]. We combined it with the solitonic description of the baryon used in [10], and we obtained analytically the mass shift of the baryons, which is linear in the pion mass squared. The slope depends on the species of the baryons.

The slope computed for nucleons (whose value is also shared with Δ), with inputs chosen to be f_π and m_ρ as in [1, 2], roughly agrees with lattice data for nucleons and Δ. We find that the slope is shared also with $N(1535)$, while the slope for the Roper excitation $N(1440)$ is found to be larger. These slopes for excited baryons are our predictions, and it would be quite interesting if they can be confirmed by lattice computations in the future. Although we work in large N_c QCD with the holographic approach, we hope that the tendency of the difference in the slope may be the same for QCD.

In Sec. 3, we gave various discussions on the vector meson mass shift, but our naive arguments showed that possible effects are subleading order in the large λ expansion (which justifies our computation in Sec. 2). It would be important to clarify this issue further and compute the vector meson mass shift more rigorously.

Acknowledgments

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